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## THE NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from May Number.]

### SCHOLIUM.

Nevertheless it might be doubted, whether, from whatever point  $K$  (assumed indeed in  $BX$  before the meeting of this  $BX$  with the other  $AX$ ) erected toward the parts of the straight  $AX$ , a perpendicular must meet this (fig. 29) in some point  $L$ ; provided of course those two, before the aforesaid meeting, are assumed ever more to approach each other mutually [and not to meet at any finite remove].

But I say it will follow completely thus.

Proof. Let there be assigned in  $BX$  any point whatever  $K$ . In  $AX$  is taken a certain  $AM$  equal to the sum of this  $BK$  and of twice  $AB$ .

Then from the point  $M$  is drawn to  $BX$  (according to Eu. I. 12) the perpendicular  $MN$ . According to the present supposition,  $MN$  will be less than  $AB$ . Wherefore  $AM$  (made equal to the sum of  $BK$  and of double  $AB$ ) will be greater than the sum of  $BK$ ,  $AB$ , and  $NM$ . Now it behooves to show this same  $AM$  to be less than the sum of  $BN$ ,  $AB$ , and  $MN$ , that thence it may follow this  $BN$  is greater than the aforesaid  $BK$ , and therefore the point  $K$  lies between the points  $B$  and  $N$ .

Join  $BM$ . The side  $AM$  will be (from Eu. I. 20) less than the two remaining sides together  $AB$  and  $BM$ . Again the side  $BM$  (from the same Eu. I. 20) will be less than the two sides together  $BN$  and  $MN$ . Therefore the side  $AM$  will be by much less than the three sides together  $AB$ ,  $BN$ , and  $NM$ . But this was to be shown, in order to deduce that the point  $K$  lies between the points  $B$  and  $N$ . Thence however it follows, that the perpendicular from the point  $K$  erected toward the parts of  $AX$  must meet this in some point  $L$  stationed between the points  $A$  and  $M$ ; else obviously (against Eu. I. 17) it must cut either  $AB$  or  $MN$  perpendiculars to  $BX$ .

Quod etc.

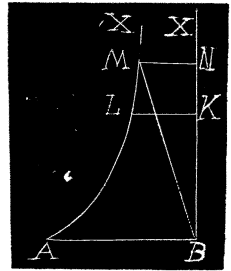


Fig. 29.

[To be Continued.]